(Minimum Homework: all odds)

#1-24 Use the Fundamental Theorem of Calculus to evaluate the definite integral.

1) $\int_2^5 (4x - 3)dx$

$$= \int_2^5 4x\,dx - \int_2^5 3\,dx$$

1st:

$\int (f(x) - g(x))dx = \int f(x)dx \ - \int g(x)dx$

$$= 4\int_2^5 x\,dx - \int_2^5 3\,dx$$

2nd: $\int af(x)dx = a\int f(x)dx$

$$= 4 \cdot \frac{1}{2} x^2 - 3x \Big|_2^5$$

$$= 2x^2 - 3x \Big|_2^5$$

3rd:

Power Rule: $\int x^n dx = \frac{1}{n+1} x^{n+1} + C \quad provided\ \overset{n}{\cancel{x}} \neq -1$

$$= 35 - 2$$

$$= 33$$

Integral of a constant Rule: $\int a dx \ = \ ax + C \quad (a\ is\ any\ real\ number)$

4th Evaluate the integral at 5 then at 2

$$\boxed{\begin{array}{l} 2(5)^2 - 3(5) = 50 - 15 = 35 \\ 2(2)^2 - 3(2) = 2 \end{array}}$$

5th subtract the results to get the answer

$answer$: 33

3) $\int_3^7 5dx$

1st: Integral of a constant Rule: $\int adx \; = \; ax + C \quad (a\ is\ any\ real\ number)$

$$= 5x \Big|_3^7$$

2nd Evaluate the integral at 7 then at 3

$5(7) = 35$

$5(3) = 15 \longrightarrow$

$$= 35 - 15$$

$$= 20$$

3rd subtract the results to get the answer

Answer: 20

5) $\int_0^3 (4x^3 + 3x^2 - 7)dx$

$$= \int_0^3 4x^3 dx + \int_0^3 3x^2 dx - \int_0^3 7 dx$$

1st:

$\int (f(x) + g(x))dx = \int f(x)dx \quad + \int g(x)dx$

$$= 4\int_0^3 x^3 dx + 3\int_0^3 x^2 dx - \int_0^3 7 dx$$

$\int (f(x) - g(x))dx = \int f(x)dx \quad - \int g(x)dx$

$$= 4 \cdot \frac{1}{4} x^4 + 3 \frac{1}{3} x^3 - 7x \Big|_0^3$$

2nd: $\int af(x)dx = a\int f(x)dx$

$$= x^4 + x^3 - 7x \Big|_0^3$$

3rd:

Power Rule: $\int x^n dx = \frac{1}{n+1} x^{n+1} + C \quad provided\ n \neq -1$

$$= 87 - 0$$

Integral of a constant Rule: $\int a dx \ = \ ax + C \quad (a\ is\ any\ real\ number)$

$$= 87$$

4th Evaluate the integral at 3 then at 0

$$(3)^4 + (3)^3 - 7(3) = 87$$

$$(0)^4 + (0)^3 - 7(0) = 0$$

5th subtract the results to get the answer

$answer$: 87

7) $\int_0^2 3e^x dx$ $= 3\int_0^2 e^x dx$

1st: $\int af(x)dx = a\int f(x)dx$ $= 3e^x \Big|_0^2$

$= 3e^2 - 3$

2nd: "e" Rule $\int e^x dx \ = e^x + C$

3rd Evaluate the integral at 2 then at 0

$3e^2$

4th subtract the results to get the answer

$3e^0 = 3(1) = 3$

$answer$: $3e^2 - 3$

9) $\int_1^e \frac{3}{x}dx$

$$= \int_1^e 3x^{-1}dx$$

1st: Rewrite with -1 exponent

$$= 3\int_1^e x^{-1}dx$$

2nd: $\int af(x)dx = a\int f(x)dx$

$$= 3\ln|x| \Big|_1^e$$

$$= 3 - 0$$

3rd: "ln" Rule: $\begin{cases} \int x^{-1}dx = \ln|x| + C \\ \int \frac{1}{x}dx = \ln|x| + C \end{cases}$

$$= 3$$

4th Evaluate the integral at e then at 1

$$\boxed{\begin{aligned} 3\ln|e| &= 3(1) = 3 \\ 3\ln|1| &= 3(0) = 0 \end{aligned}}$$

5th subtract the results to get the answer

answer: 3

11) $\int_1^e 7x^{-1}dx$ $= 7\int_1^e x^{-1}dx$

$= 7\ln|x| \Big|_1^e$

1st : $\int af(x)dx = a\int f(x)dx$

2nd : "ln" Rule: $\begin{cases} \int x^{-1}dx = \ln|x| + C \\ \int \frac{1}{x}dx = \ln|x| + C \end{cases}$ $= 7 - 0$

$= 7$

3rd Evaluate the integral at e then at 1

$\left.\begin{array}{l} 7\ln|e| = 7\cdot 1 = 7 \\ 7\ln|1| = 7\cdot 0 = 0 \end{array}\right\}$

4th subtract the results to get the answer

answer: 7

13) $\int_1^2 3(3x+1)^2 dx$ $\quad = \int (3x+1)^2 \cdot 3dx$

let $u = 3x+1$

Rewrite the problem so that the parenthesis is first: $\quad \int u^2 \cdot 3dx$

Next: $let\ u = inside\ of\ the\ parenthesis$ $\quad = \int u^2 \cdot du$

Rewrite the problem so that the "parenthesis is changed to an "u"

$u = 3x+1$ $\quad = \frac{1}{3}u^3$

Next find $\frac{du}{dx}$ $\quad dx\frac{du}{dx} = 3dx$

$du = 3dx$ $\quad = \frac{1}{3}(3x+1)^3\Big|_1^2$

Multiply by dx to clear the fraction.

Next replace to make problem only have u's $\quad = \frac{343}{3} - \frac{64}{3}$

Next integrate: use Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ \cancel{x}\ n \neq -1$ $\quad = \frac{279}{3}$

Last change $u\ back\ to\ compute\ the\ integral$ $\quad = 93$

$$\boxed{\begin{aligned} \frac{1}{3}(3(2)+1)^3 &= \frac{343}{3} \\ \frac{1}{3}(3(1)+1)^3 &= \frac{64}{3} \end{aligned}}$$

Evaluate the integral at 2 then at 1

subtract the results to get the answer

$answer$ 93

15) $\int_1^2 9(3x+1)^2\,dx$ $= \int (3x+1)^2\, 9\,dx$

Rewrite the problem so that the parenthesis is first:

let $u = 3x+1$

Next: $let\ u = inside\ of\ the\ parenthesis$

$= \int u^2\, 9\,dx$

Rewrite the problem so that the “parenthesis is changed to an "u"

$u = 3x+1$

Next find $\frac{du}{dx}$ $dx\,\frac{du}{dx} = 3\,dx$

Multiply by dx to clear the fraction. $3\,du = 3\cdot 3\,dx$ $= \int u^2 \cdot 3\,du$

Not good enough, multiply to make a perfect match

$3\,du = 9\,dx$ $= 3\int u^2\,du$

Next replace to make problem only have u’s

$= 3\cdot\frac{1}{3}u^3$

Next integrate: use Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C\ \ provided\ \cancel{x}\ n \neq -1$

$= 1u^3$

Last change $u\ back\ to\ compute\ the\ integral$

$= (3x+1)^3 \Big|_1^2$

Evaluate the integral at 2 then at 1

$\boxed{\begin{array}{l}(3(2)+1)^3 = 343 \\ (3(1)+1)^3 = 64\end{array}}$

subtract the results to get the answer

$= 343 - 64$

$= 279$

$answer$: 279

17) $\int_{-2}^{4}(2x)\,(x^2-1)^2dx$ $= \int (x^2-1)^2 \cdot 2x\,dx$

Rewrite the problem so that the parenthesis is first: let $u = x^2-1$

Next: $let\ u = inside\ of\ the\ parenthesis$ $= \int u^2 \cdot 2x\,dx$

Rewrite the problem so that the “parenthesis is changed to an "u"

$u = x^2-1$

Next find $\frac{du}{dx}$ $\cancel{dx}\frac{du}{\cancel{d}x} = 2x\,dx$

Multiply by dx to clear the fraction. $du = 2x\,dx$ $= \int u^2\,du$

Next replace to make problem only have u’s $= \frac{1}{3}u^3$

Next integrate: use Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C\ \ provided\ \overset{n}{\cancel{x}} \neq -1$

$= \frac{1}{3}(x^2-1)^3 \Big|_{-2}^{4}$

Last change $u\ back\ to\ compute\ the\ integral$

Evaluate the integral at 4 then at -2

$\boxed{\begin{array}{l} \frac{1}{3}(4^2-1)^3 = \frac{1}{3}(15)^3 = 1125 \\ \frac{1}{3}((-2)^2-1) = \frac{1}{3}(3)^3 = 9 \end{array}} \rightarrow = 1125 - 9 = 1116$

subtract the results to get the answer

$answer$: 1116

19) $\int_{-2}^{4}(6x)\,(x^2-1)^2dx$ $= \int (x^2-1)^2 6x\,dx$

let $u = x^2-1$

Rewrite the problem so that the parenthesis is first:

$\int u^2 6x\,dx$

Next: $let\ u = inside\ of\ the\ parenthesis$

Rewrite the problem so that the "parenthesis is changed to an "u" $= \int u^2 \cdot 3du$

$u = x^2-1$

Next find $\frac{du}{dx}$

$dx\frac{du}{dx} = 2x\,dx$ $= 3\int u^2 du$

Multiply by dx to clear the fraction.

$3du = 3\cdot 2x\,dx$

$3du = 6x\,dx$ $= 3\cdot\frac{1}{3}u^3$

Not good enough, multiply to make a perfect match

$= u^3$

Next replace to make problem only have u's

$= (x^2-1)^3 \Big|_{-2}^{4}$

Next integrate: use Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

$= 3375-27$

$= 3348$

Last change $u\ back\ to\ compute\ the\ integral$

$(4^2-1)^3 = 3375$

Evaluate the integral at 4 then at -2

$((-2)^2-1)^3 = 27$

subtract the results to get the answer

$answer$: 3348

21) $\int_0^1 3x^2 e^{x^3}\,dx$ $= \int e^{x^3}\, 3x^2 \cdot dx$

Rewrite the problem so that the "e" is written first: let $u = x^3$

Next: $let\ u = exponent\ of\ the\ e$ $= \int e^{u}\, 3x^2\, dx$

Rewrite the problem so that the exponent is changed to an "u" $\int e^u\, du$

$u = x^3$

Next find $\frac{du}{dx}$ $dx\,\frac{du}{dx} = 3x^2\,dx$

Multiply by dx to clear the fraction. $du = 3x^2\,dx$ $= e^u$

Next replace to make problem only have u's $= e^{x^3}\Big|_0^1$

Next integrate: "e" Rule $\int e^x dx \ = e^x + C$ $= e - 1$

Last change $u\ back\ to\ calculate\ the\ integral$

$e^{(1)^3} = e^1 = e$

Evaluate the integral at 1 then at 0

$e^{(0)^3} = e^0 = 1$

subtract the results to get the answer

answer: $e - 1$

23) $\int_0^1 6x^2 e^{x^3} dx$ $= \int e^{x^3} 6x^2 dx$

Rewrite the problem so that the "e" is written first: let $u = x^3$

$= \int e^u 6x^2 dx$

Next: $let\ u = exponent\ of\ the\ e$

$= \int e^u 2du$

Rewrite the problem so that the exponent is changed to an "u"

$u = x^3$

Next find $\frac{du}{dx}$ $dx\frac{du}{dx} = 3x^2 dx$

$= 2\int e^u du$

Multiply by dx to clear the fraction. $2du = 2 \cdot 3x^2dx$

$2du = 6x^2dx$

Not good enough, multiply to make a perfect match

$= 2e^u$

Next replace to make problem only have u's

$= 2e^{x^3}\Big|_0^1$

Next integrate: "e" Rule $\int e^x dx \ = e^x + C$

$= 2e - 2$

Last change $u\ back\ to\ calculate\ the\ integral$

$2e^{(1)^3} = 2e^1 = 2e$

Evaluate the integral at 1 then at 0

$2e^{(0)^3} = 2e^0 = 2 \cdot 1 = 2$

subtract the results to get the answer

$answer$: $2e - 2$